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Debt level and company efficiency: independence or implication? An evaluation of fuzzy implications

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Abstract. – This paper suggests a way of measuring the strength of an implication to evaluate the influence of the financing structure chosen by company on its efficiency and profitability. The first part describes the semantics of the fuzzy implication applied to continuous variables. The results of three single implication measurements between two criteria are then analyzed on a sample population of 140 companies.

Introduction

The existence of a financing structure optimized between equity and debt has been the subject of a vast debate for several decades. The issue is important, because it appears obvious that if such a structure exists, companies should strive to achieve it and, if possible, adhere to it for financing their activity. The many publications devoted to the subject, starting with the work of Modigliani and Miller (1958, 1963) (MM) do not always make it clear which if either of the two financing sources should be privileged.

From a theoretical standpoint, according to MM's theory, the value of a company is independent of its financing structure. In other words, there is no optimal debt ratio. In an ideal market, the debt level appears to have no impact on the share value (Guyon, 1992). However, whenever one of the ideal market hypotheses is relaxed, this conclusion is modified. For instance, if companies can borrow at a lower rate than investors, it is in the interest of the latter for the companies to increase their debt level. Ultimately, there are three main reason why a company does not reach its highest debt level: the influence of tax law, through the savings on taxes afforded by interest expenses for an indebted company (De Angelo and Masulis, 1980), the costs of bankruptcy (Altman, 1984) and agency costs (Jensen and Meckling, 1976; Charreaux, 1987).

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Observation of the way companies behave in the real world as regards funding shows how widely debt levels vary from one company to another. For instance, how to explain the fact that the long-term debt level:

$$\frac{\text{Long-term debt}}{\text{Equity}}$$

reported in 1996 by two French companies, each with around a thousand employees, was only 4.70% for the first but 49.37% for the second?

From the standpoint of the sector of activity, it could be expected that companies with similar activity would have similar financial profiles. But many empirical studies show the contrary to be true (Dubois, 1985). In some cases, the results of these empirical studies lead to rather problematic conclusions: “Profitability (often) appears as the main factor explaining the financing structure of companies: the more profitable the company, the lower its debt level” (Desbrières and Dumontier, 1989).

In this paper, we propose to evaluate the influence of the financing structure of a company on its efficiency and profitability. The activity of a company is conditioned by all the means available to it (equity and debt) and which are put to use (fixed assets and need for working capital) in order to realize sufficient profits, if possible, to remunerate all the capital contributors. The diagram in Figure 1 summarizes the activity process:

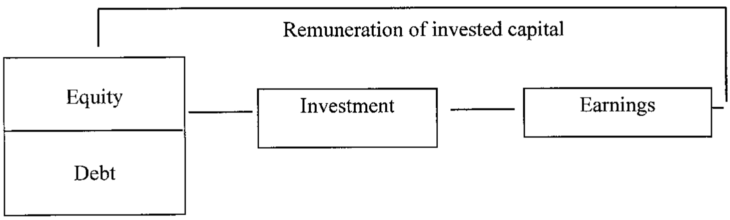

Fig. 1. *Illustration of invested capital remuneration process.*

This study, conducted per sector, is aimed at measuring the degree to which the financing structure of a company is implicated in its efficiency and profitability.

First the semantics of the implication applied to continuous variables is presented. The results of three implication measurements are then analyzed on a sample population of 140 companies. The results show in particular that it is difficult to conclude that the source of financing is neutral with respect to profitability.

1. Fuzzy implications between continuous variables

The construction of functions (*t*-norms, *t*-conorms, implications, etc.) in fuzzy logic requires all the variables used to be contained in the interval [0, 1] to be able to account for all situations ranging from 0 (false) to 1 (true).

Confronted with raw data, the expert must first solve the problem of how to transform or encode the data before choosing a family of fuzzy operators to measure the implication between criteria.

1.1. *Data encoding*

For each variable *V* with values in the interval [*m, M*], encoding consists of determining an interval [*a, b*] included in [*m, M*] such that the expert considers that:

– any value below *a*, estimated low, is encoded as 0, and

– any value above *b*, estimated high, is encoded as 1.

The new variable encoded in this way gives a measure of the strength of *V* on a weak-to-strong scale.

For linear encoding, the process is as follows:

$$\text{For } x \in [m, M] \text{ encoding of } x = \begin{cases} 0 \text{ if } x \le a \\ \varphi\left(\dfrac{x - z}{b - a}\right) \text{ if } a \le x \le b \\ 1 \text{ if } x \ge b \end{cases}$$

where φ is either linear or linear by parts (for instance by requiring the average to be encoded as 0.5).

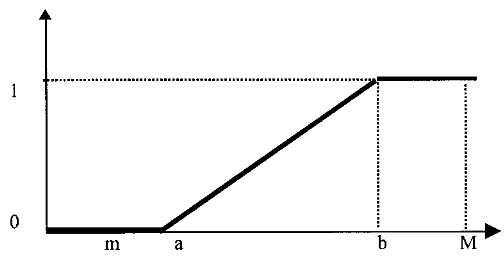

Fig. 2. *Linear encoding.*

1.2. *Fuzzy implications*

Let us consider two encoded variables, *X* and *Y.*

According to Klir and Yuan's theorem (1995): Let φ be a function defined on the interval [0,1] and verifying the following properties:

– $\varphi(0) = 0$

– φ is continuous and strictly increasing.

We can then define the negation and implication operators:

– $\text{not}\,(x) = \varphi^{-1}\,[1 - \varphi\,(x)]$

– $\text{imp}\,(x, y) = \inf\,\{1, \varphi^{-1}\,[\,\varphi\,(1) - \varphi\,(x) + \varphi\,(y)]\}$.

Taking $\varphi(x) = x$ as a special case, these operators become:

$$\text{(I)} \quad \begin{cases} \text{not}\,(x) = 1 - x \quad (\text{or } x') \\ \text{imp}\,(x, y) = \inf\,(1, 1 - x + y) = \begin{cases} 1 \text{ if } y \le x \\ 1 - x + y \text{ if } y \ge x \end{cases} \end{cases}$$

System (I) then yields the conventional negation and implication operators

$$\text{(II)} \quad \begin{cases} \text{not}(x) = 1 - x \\ \text{imp}(x, y) = \begin{cases} 1 \text{ if } y \geq x \\ \text{else } 0 \end{cases} \end{cases}$$

Operators (II) are a degeneration of operators (I).

With two variables, the diagrams in Figure 3 indicate the four possible implications, for which the rejection regions are cross-hatched (see Fig. 3).

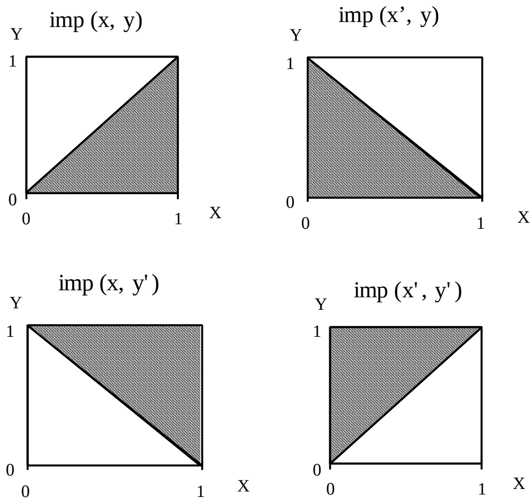

Fig. 3. *Rejection regions for x in [0,1].*

The measure of the strength of an implication, *e.g.* imp (x, y) based on a sample $[(x_1, y_1), \ldots (x_n, y_n)]$ is as follows:

$$m[\text{ imp }(x, y)] = \frac{\text{card } D}{n}$$

Where D denotes all the (x_i, y_i) pairs in the acceptance region.

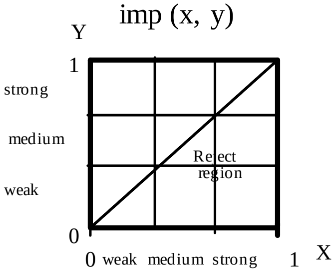

***Fig. 4.**. Rejection region for three classes.*

1.3. *Semantics of the implication*

Let us consider the implication imp (*x*, *y*).

A measure of imp (*x*, *y*) equal or close to 1 is interpreted as follows:

– in a region where *X* is weak, *Y* can have any value;

– if *X* is medium, *Y* can only be medium or strong;

– if *X* is strong, *Y* is always strong.

An equivalent way of stating this would be to say that a weak *Y* cannot be obtained by a weak *X*.

1.4. *Validation of the implication*

The validation can be construction using the test suggested by Briand (1995) and Gras (1996).

Where *n* is the sample size, *x* is the number of items verifying character *X*, $\bar{n} = n - x$, *y* is the number of items verifying character Y, $\bar{y} = n - y$.

If $n_y \neq n$, the measure of the $X \rightarrow Y$ implication, mes($X \rightarrow Y$), is:

$$\text{mes}\ (X \rightarrow Y) = \frac{1}{\sqrt{2\pi}} \int_{\text{mes}\ (X \rightarrow Y)}^{+\infty} e^{\frac{-t^2}{2}}\, d\dot{t}.$$

2. Application results

2.1. *Sample*

We tested this implication measure on sample population of French companies in the metalworking economic sector of activity for 1995 (Sector 28 of the INSEE classification; the sample was obtained from the "Diane" database of SCRL). This sample population included 140 companies of different legal forms of association and sizes out of a total population of 6022 companies in this sector of activity, which includes boilermaking, general mechanics, metalworking, production of metal components for the construction industry, etc.

We calculated the 10 ratios listed in Table 1 for all the companies in the sample. No processing or limiting was applied to these ratios. It should be stressed that this choice meant that we accepted from the outset the risk of a very wide dispersion in some cases.

It can be seen that ratios 2, 6, 7 and 8 give an idea of the nature and structure of the financing of the companies in the sample as well as their debt type and level. Ratios 1 and 3 provide measures of the productivity of labor and capital. The economic profitability of the companies (ratio 4) is calculated before tax and nonrecurring costs. The operating cash flow is the expression of the gross profit generated by the companies and ratio 10 is the resulting gross margin. Finally, ratio 9 is the conventional formulation of financial profitability with respect to equity.

Table 2 summarizes the main features: mean, standard deviation and scatter of the ten ratios analyzed. For most of them, it exhibits extremely wide variations between the extreme values observed. It should be noted that these extreme values concern only a small number of companies in each case, generally one or two, but not the same one or

TABLE 1. List of ratios used.

1 – $\frac{\text{Added value}}{\text{Manpower}}$	6 – $\frac{\text{Total debt}}{\text{Equity}}$
2 – $\frac{\text{Long and medium term debt}}{\text{Short term debt}}$	7 – $\frac{\text{Long and medium term debt}}{\text{Equity}}$
3 – $\frac{\text{Added value}}{\text{Fixed assets}}$	8 – $\frac{\text{Long term capital}}{\text{Balance sheet total}}$
4 – $\frac{\text{Operating income}}{\text{Balance sheet total}}$	9 – $\frac{\text{Net earnings}}{\text{Equity}}$
5 – $\frac{\text{Payroll costs}}{\text{Added value}}$	10 – $\frac{\text{Operating cash flow}}{\text{Sales}}$

TABLE 2. Mean and scatter of the ratios.

Ratios	**Minimum**	**Maximum**	**Mean**	**Std deviation**
1	61.8	593.73	240.17	66.71
2	0	9.86	0.26	0.86
3	0.08	29.05	4.57	4.53
4	–0.21	0.24	0.06	0.09
5	0.42	1.92	0.82	0.20
6	–55.3	93.57	4.60	13.19
7	–0.59	2.38	0.59	0.39
8	–0.26	1	0.60	0.32
9	–7.18	7.94	0.16	1.25
10	–0.29	0.28	0.06	0.08

two from one ratio to another. Observation of the data shows that around twenty companies have a least one ratio whose value is more than three standard deviations from the mean value of the sample.

2.2. *Results*

The results given below concern only single relationships between two criteria.

Three implication measures were tested. The first concerns the alleged relationship between labor productivity (ratio 1) and the profitability of the company's activity. The economic profitability (ratio 4) was privileged for this first test. The other two measures concern the relationship between financing structure and earnings.

Figure 5 is an illustration of the productivity-profitability relationship.

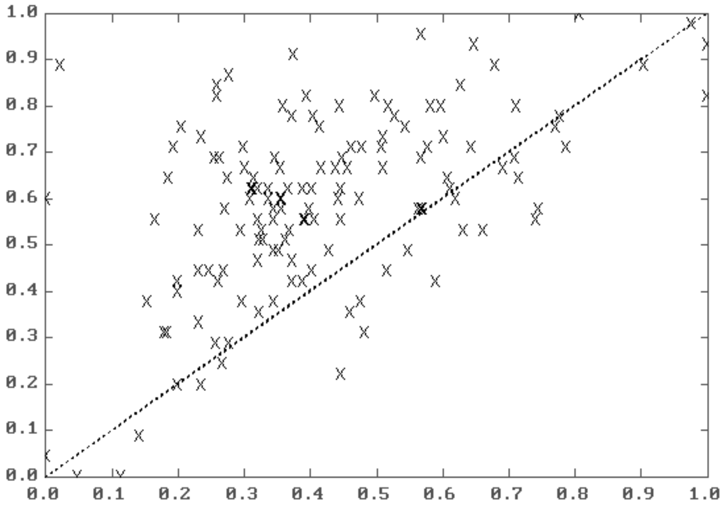

Fig. 5. *Productivity $\Longrightarrow$ profitability implication.*

This relationship can be expressed very simply for all the companies in the sample:

– high productivity implies high profitability;

– medium productivity implies medium or high profitability;

– low productivity does not imply any level of profitability.

Another way of reading the same results is to say that low profitability can only result from low productivity.

The valuation of this first implication is 0.94.

As mentioned above, the financing structure of the companies was evaluated from four ratios. For this initial approach, we privileged ratio 6:

$$\frac{\text{Total debt}}{\text{Equity}},$$

often called debt leverage (long-term debt is sometimes used in this ratio instead of total debt). This criterion was applied to both the economic and financial profitability (ratios 4 and 9) achieved by the companies.

Figures 6 and 7 illustrate the leverage effect on the economic profitability and the financial profitability respectively.

First of all, the fairly similar appearance of the two clusters of points should be noted. These two clusters can be interpreted as follows:

– strong leverage implies medium (economic and financial) profitability;

– medium leverage implies medium-low to medium-high profitability;

– low leverage does not imply any level of profitability.

These results are of course specific to the sector analyzed. However, the identical valuation of 0.95 obtained for both implications clearly shows that the financing structure chosen (or supported) by the companies in this sector has a definite influence on both their economic and their financial profitability.

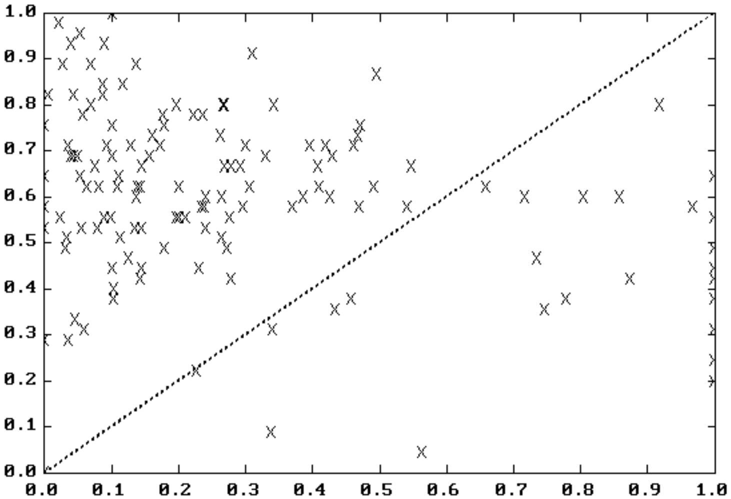

Fig. 6. *Leverage* $\Rightarrow$ *economic profitability implication.*

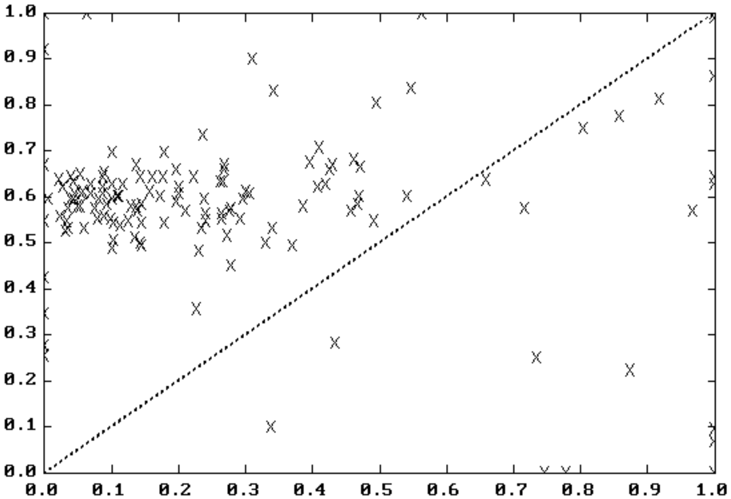

Fig. 7. *Leverage* $\Rightarrow$ *financial profitability implication.*

Other single implications were also tested. For instance, the implication between the productivity of capital (ratio 3) and economic profitability was valued at 0.7, which can also be considered strong. However, the fact that it is not close to 1 can probably be explained by the somewhat heterogeneous character of the term fixed assets.

Conclusion

These initial strength measurements of single implications between two criteria allowed us to verify that the fuzzy implication operator proposed in the first part of the paper was operational. In particular, they allow us to conclude that, for the sector analyzed, both economic and financial profitability of the companies were strongly dependent on the financing structures adopted.

From a practical standpoint, our work could be continued in two ways. The first is to apply the operator to other sample populations from different sectors of economic activity and the second is to supplement the analysis by measuring the implication strength between several criteria considered simultaneously.

Finally, from a theoretical standpoint, the point we feel should be privileged concerns analysis of the impact of the different ways of encoding the implication measures.

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