

AN EVOLUTIONARY MODEL OF VOTING

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Abstract. Collective allocation of resources that takes place in political markets is characterized by the complex exchange that emerges among the individuals involved. Traditional Public Choice models depart from individual rational choice in a setup in which many of its strict requirements need not hold. This paper introduces a model of social interaction among agents in a simple political market which departs from bounded rationality and evolutionary dynamics as the key mechanisms that drive individual behavior. Learning plays a significant role as it allows to establish an individual link between decisions and collective outcomes. The model is that of a representative democracy with two parties in which individuals are restricted to a one dimensional policy space. The main findings from computational experiments allow us to revise the results of traditional models, specially those related to the *voting paradox*. We find that turnout levels may be higher than expected in a population composed of fully rational agents, and that there is a rationale for abstention that stresses the role of limited information, the discounting of the future, and the extent of the redistributive policies.

Classification Codes. D72.

1. INTRODUCTION

Even though rational choice, as a unifying paradigm in the social sciences, has been successful in the explanation of some economic and social phenomena, it also has been at odds in explaining irregular features or situations, which has led to an ongoing acceptance of alternative explanations whenever the extreme rationality assumption yields trivial or counter-factual outcomes. The complexity

Keywords and phrases: voting, vote motive, collective action, evolution, bounded rationality, learning, classifier systems.

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of some strategic interactions² has been tackled, with a different degree of success, by developments in the fields of bounded rationality, evolutionary economics and evolutionary game theory³. While the influence of these developments in market exchange processes has been significant, there is a lack of formal work addressing political exchange. And it is here where we find some of the predictions of the rational choice theory to be systematically violated.

Bounded rationality, and the evolution and learning of agents in social settings are specially suited for explaining political behavior. And this is so as individuals engaged in collective decision making may have less incentives for being unboundedly rational than those in market exchange, at least because of two reasons: first, benefits from market exchange are clearly identified and perceived by the exchanging parties, while those of the political exchange may be more difficult to identify and allocate; second, contrary to what happens in the market, there is a weak relation between individual decisions and the outcomes of political processes⁴. Broadly speaking these suggests that models based on unbounded rationality assumptions may have difficulties in explaining non-market behavior of individuals. However most public choice models depart from the orthodox maximizing behavior of an omniscient agent subject to constraints; given these assumptions paradoxical outcomes are to be found. In case of the standard electoral model the *voting paradox* arises. Briefly speaking the standard model departs from the basic *downsian* (Downs 1957, or Ryker and Ordeshook 1968) assumption. The rational voter hypothesis. Following it, individuals undertake a cost-benefit analysis when voting. If political participation is costly and the probability of an individual vote affecting the outcome of an election is marginal, then voting may be represented as a public good. As in any public good setting, rational agents will tend to free ride on the political participation of others, as there are no incentives for voting since gathering (and processing) information about different electoral programs is a costly activity and the expected benefits are small. Consequently individuals would tend to abstention. Related to the rational voter hypothesis, we may expect that a rational voter guided by expectations about the future and not past information. Indeed adaptive behavior is clearly in contradiction with rationality, and voters should somehow include the future when forming their decisions.

²Complexity that arises because the decision making process is itself complex, or because the interaction set is complex. While the former refers to many market exchange situations in which decision making takes place as forward looking agents collect information about future events with uncertainty, the latter will be related to the main point of this paper: collective decision making is complex because the exchange involves a set of $N > 2$ agents and individual actions have a unclear effect on the outcomes and there is an interrelation between individual decisions. We will return to this later.

³Some overviews on different aspects of bounded rationality and evolution, applied to economics and game theory are Conlisk (1996), Mailath (1998) and Nelson (1995).

⁴Many authors in the public choice literature recognize the weak link between actions and outcomes in collective action. This led to early criticisms to the indiscriminate application of instrumental rationality to the public domain. As an example, Buchanan & Tullock (1962) point out to the weak relation between individual decisions and collective outcomes to be a limit to individual rationality, developing the concept of *rational ignorance*, that shares much in common with models of bounded rationality.

However, empirical observations contradict the rational voter hypothesis. Participation is much higher than the predicted level. People does not free-ride, or at least not as much as expected, on others participation. And empirical models in which voters use readily available information, usually past information about the macroeconomic performance such as employment or inflation rates or GDP growth, outperform more sophisticated models of voter behavior. These models of political action based on adaptive voters, sometimes called *the retrospective voter model*, introduce learning as a basic element⁵ in clear contrast with rational voter models. Recently Swank (1998) rationalizes adaptive behavior in an attempt to maintain the instrumental consistency of the agents. In summary we may conclude that rationality is at odds with voting behavior.

To solve this apparent contradiction, models include irrational behavior, emotions, social norms... among other as behavioral *pathologies*⁶, or modify the setting maintaining the basic rationality hypothesis⁷. While public choice has introduced some new elements in the rational choice debate, it is still dependent on the basic behavioral hypothesis of microeconomics.

In this paper we analyze political action from an economic perspective using boundedly rational individuals within an evolutionary framework. What we propose is to change the perspective by using an agent based modeling framework and analyze the emergent properties at the macro-political level. To this end we will develop a model of political participation in which individuals use simple rules of thumb in making their political decisions under different scenarios; the model tries to reflect the continuous process of interaction among agents in the political market leads to a knowledge accumulation and spread, and to the evolution of the social system. The paper is structured as follows: in Section 2 we introduce the theoretical setup; Section 3 shows the results of some computational experiments; finally some conclusions will be discussed in Section 4.

2. AN AGENT-BASED MODEL OF NON-MARKET DECISION MAKING

Next we present a basic model of collective choice in which individual behavior evolves as the result of a learning process.

2.1. THE POLITICAL ECONOMY

Consider an economy in which N agents interact during T time periods. Individuals allocate their resources between private and public activities; the latter implies that a mechanism for the allocation of collective resources is needed. We consider a political mechanism based on a representative democracy in which two parties (R and D) compete for votes in a one dimensional issue space: R offers a

⁵See Hibbs (1977) or see Alesina & Rosenthal (1995) for a survey

⁶See Mueller (2000) for a revision of such concepts.

⁷Different models (see Ledyard, 1984; Palfrey & Rosenthal, 1984; 1985; 1987 or more recently Campbell, 1999) and Shachar and Nalebuff (1999), apply the *downsian* cost-benefit calculus, and, with slight variations, all reach to the same core of conclusions.

low level of public redistribution while D offers a higher collectivization of private income. The winning party, *i.e.* the one which casts the higher number of votes, defines the political action to be taken, which will be a negative income tax system (NIT). Agents may freely chose either to vote any party or not to vote. Their choices will be driven by the shaping of their political preferences by a process of cultural evolution.

In the political process each party, R and D, offers a tax rate, t_D and t_R . Once elections take place, a tax rate $t^* \in \{t_D, t_R\}$ is chosen. Let y_t^i be agent i 's income at time t , then the total amount of taxes raised is $S = t^* \sum y_t^i$. This will be collectively used for redistributive purposes: the NIT system implies that agent i net transfers to/from the public sector will be:

$$T_t^i = -S + t^* y_t^i \quad (1)$$

which modifies agents' net income.

2.2. THE AGENTS

Agents are the basic building block of our model. We model an agent by her internal information, and by a set of rules. An agent holds information about: (1) her state, which in the model is given by the level of income (low \underline{y} or high \bar{y}); (2) the fitness of any available political option⁸ in any given state. Agents use their internal information to interact in the political market by using rules. A rule is a mapping from a given state to a political decision: analyzing available options and their fitness, agents take next action. This simple idea may be formally developed by using the concept of *classifier systems*. Formally defined, a *classifier system* is a learning system in which a set of rules compete one with each other for the right of performing an action in a given state: the most fitted rule for a given state will be chosen to perform its corresponding action⁹. Define the *fitness* of a political option as an ordinal value that the agent assigns to an action given the state of nature. Define a classifier as the pair of an option and its related fitness: $\langle \text{option}, \text{fitness} \rangle$. Then a classifier system will be the set of all possible pairs.

A simple example will clarify the point: for a given state, which in our example is high income/low income, an agent has to decide the allocation of her resources between private or public activities. The final effect of public outcomes, which arise as the result of the collective decision making process, will be to affect private decisions by modifying an individual's budget constraint. Hence we may state the problem in the following terms: given a state of nature, an agent has to choose a political option. The logic underlying the actual setup is that an agent will choose the most fitted rule given the current state. Call f_t^k the fitness of

⁸Which may be any action taken in the political process: either being to vote or not to vote, and if the former which party.

⁹Applications in economics include the emergence of a medium of exchange in an artificial economy (Marimon *et al.*, 1990), analyzing dynamic programming problems from the perspective of rules of thumb, Lettau & Uhlig (1999), exploring the role of learning by imitation, Basçi (1999), or the differences between individual and social learning in Vriend (2000), to mention some.

rule k at moment t ; then individual will choose the political option associated with rule k iff $f_t^k > f_t^i \quad \forall i$ ¹⁰. Note that any political option is available at any given state. However there is a link between states and actions, as for each state there is a different classifier. Hence actions fitted for one state, *i.e.* an action that would be chosen in a low income situation, will not necessarily be fitted for the other, as rules have different strengths for different states. Finally notice that individuals start with a (perhaps inherited) given system of rules. It is from the collective interaction, that the fitness of these rules evolve. How this happens will be explained next.

2.3. THE EVOLUTIONARY MECHANISM

Evolution takes place as individuals learn about the outcomes of the political process and the different political options. The process of cultural evolution influences political outcomes and is influenced by the economic performance of the agents. It can be described as an *evolutionary voting game*; next we discuss its main features.

At the beginning of period t , each agent j is endowed with period t income y_t^j . Income follows a *Markov process* with 2 different states: high income (\bar{y}) and low income(\underline{y}). The probability of transition from state i to state j is:

$$p_{ij} = \begin{cases} \frac{(1-\rho)}{2} & i \neq j \\ \rho + \frac{(1-\rho)}{2} & i = j \end{cases} \quad (2)$$

where $\rho \in [0, 1]$ is the probability of remaining in the same state at two correlative time periods. Additionally, while remaining at one state, income follows a random walk:

$$y_t^i = y_{t-1}^i + u_t^i \quad \text{where } u_t^i \text{ is } i.i.d.(0, \sigma). \quad (3)$$

Once income has been determined, collective decisions take place. Agents vote for a proportional tax system¹¹ which will modify her initial endowment by the net transfers from (towards) the public sector. The instantaneous welfare of an agent is given by her net income available for purchasing a private good. In this case, if we rule out wealth accumulation, her welfare will be q_t^j which is the quantity of private good consumed by agent j , at moment t given her income plus (minus) government transfers.

¹⁰Two comments apply: first in case the inequality is not strict a random choice will solve the ties; second, in the model we introduce a *trembling hand-effect*: there is a slight probability π for the agents to select the wrong action, which allows the model of being partially independent of the initial state.

¹¹Restrictions to the minimum/maximum level of redistribution, as well as the decision making rules are considered to be constitutionally fixed.

At the end of every sub-game the state changes and agents update their classifier systems. In doing so, they consider not only the instantaneous welfare from consumption, but also the future state that is induced. Let k be the winning political rule, and i be the most fitted rule for the new state, given the classifier at t . Then call f_t^k and f_t^i their respective fitness. The updating of the classifier is done in two steps.

First individuals update the fitness of the winning political option, which is done in the following way

$$f_{t+1}^k = f_t^k + \psi \left(q_t^j + \beta f_t^i - f_t^k \right) \quad (4)$$

being ψ_i a decreasing weighting sequence¹². It is worth noting that expression (4) not only takes into account the current consumption, but links it with the future by including the state induced in the next step, discounted at a rate β , which describes the temporal preferences of the agents. Obviously only winning rules are updated, as there is no other way to update a political option but with the experience of individuals once it is applied as a political program.

Second, agents' update the fitness of abstention at each stage, which depends on a comparison of the utilities from alternative programs. Let f_t^0 be the fitness of abstention, then:

$$f_{t+1}^0 = f_t^0 + \psi \left(w_t(q_t^j - \hat{q}_t) + \beta f_t^i - f_t^0 \right). \quad (5)$$

Being \hat{q} the alternative level of private good, in case other political option was the winning one, and w_t is the level of abstention. In the case in which no agent in the economy votes, one vote would be decisive and individuals will be facing a high opportunity cost. Obviously this cost decreases as turnout levels are higher, and in the limit (when turnout is 100%) becomes null.

This updating leads to a new ranking of political options that will be used by the agent when taking decisions in the future, ensuring a process of evolution and learning. Let us summarize the basic functioning of the model in an algorithmic way.

- (i) At stage t each voter selects rule with the highest strength for a given state (high income/low income). Let it be rule k and f^k its associated strength.
- (ii) Each agent performs the action associated to the previously selected rule: vote D, vote R, don't vote. As a result of the electoral process, there is a redistributive flow. Let q^j define the net income after the NIT is applied. This is to be understood as the instantaneous individual utility of the policy of the winner party.
- (iii) At stage $t + 1$ the state of each voter changes with a given probability. Let now be rule i , with strength f^i the most fitted. At this point voters update

¹²In our case, we used $\psi = \frac{1}{s}$, being s the number of times that rule j has been activated. It ensures that the agents corrections over time will finally dampen and set to a steady state.

the classifier following expressions (4) and (5). And the new classifier system will be used for taking political decisions. We go back to (i).

3. COMPUTATIONAL EXPERIMENTS

In order to investigate the main features of the model previously described, we conducted three different simulation experiments. Unless otherwise indicated, we simulated a political economy with a stationary population of $N = 1000$ agents which interacted during $T = 1000$ time periods. At the beginning of each interaction the endowment of each agent was fixed, and hence the state, according to expressions (2) and (3). With respect to income there are two additional features to be considered. First, the ratio between high income/low income has been considered fixed and equal to $\bar{y}/\underline{y} = 4$. Second, the probability of being in the same state ρ affects individual well-being and will be used for the purpose of analyzing the effects of state changes, to be understood as economic changes, over political outcomes.

We run different experiments to test the implications of modeling boundedly rational agents and compare its results with the conventional literature. We group them in two subsections: the first will deal with the rationale of voting; the second will analyze the outcomes of political participation in terms of the redistribution that agents aim.

3.1. THE VOTE MOTIVE REVISITED

The first question we want to answer is which parameters affect the turnout level of an election. According to this we expect that turnout will be affected by three basic parameters of the experiments: (1) the constituency size, (2) the discounting level, and (3) the individual benefits of the redistributive policy.

Starting with (1), let us assume that an agent is perfectly informed; then the probability of affecting the outcome of an election diminishes with the constituency size¹³. To test this hypothesis we simulated the evolution of different sized populations; to this end we re-seeded the population with the same random seed thus at least the common individuals and the environment of every population were identical¹⁴. Figure 1 shows the effect of modifying the constituency size in the original population. Apart from the lowest size (50 individuals), once the population reached a given threshold (about 100 agents), simulations supported the conclusion that participation tends to decrease with the population size. For smaller populations (up to 100 individuals), simulations did not support the hypothesis and in fact a counter-intuitive outcome emerged from them.

Next we investigate the link between discount rate and participation. As it was already mentioned rational agents take expectations, and not past actions, as

¹³For instance, see calculation for US elections in Shachar and Nalebuff (1999).

¹⁴Except were indicated we proceeded in the same way: every time we wanted to analyze the effect of one parameter over the population we only modified this parameter thus allowing for the same population to be reproduced and maintaining a type of *ceteris paribus* hypothesis.

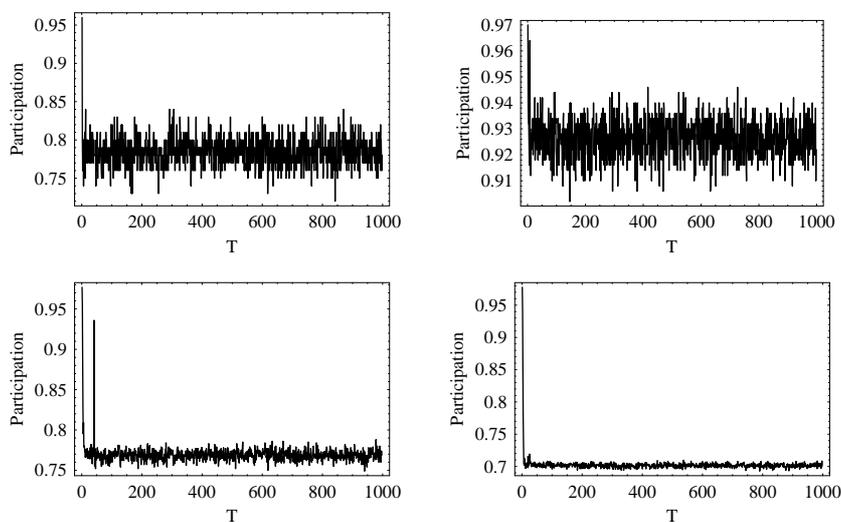


FIGURE 1. Effect of the constituency size on the turnout level. From left to right and from top to bottom, population size N is 50, 100, 1000 and 5000.

a guide of behavior. Discounting allows for taking into consideration the effects of individual political decisions in the future. Current collective choice outcomes shape classifier systems of agents, hence affecting future individual decisions and, although in a weak manner, the outcomes of future voting processes. As the proposed model includes the future, we want to test which is the role it plays in the individual decisions. We conjecture that as in the cost-benefit model, the effects of any vote in the future state of an individual is negligible as, again, the probability of one vote affecting the outcomes (present or future) is low. Hence turnout shall be negatively related to discounting. We run two different simulations for experimentally testing this hypothesis. In the first, we replicated the same population with the same initial conditions, modifying the discount rate and the economic stability of the system¹⁵. Results of this experiment are shown in Figure 2, which shows the fitted curve corresponding to a quadratic relationship between turnout and discount rate; by using least squares we adjusted the experimental points for the three simulations yielding the theoretical relations that are shown in the graph. The estimated relationships were:

$$x = \alpha_0 + \alpha_1 \beta^2. \quad (6)$$

Being x turnout, and α_1 the parameter that determines the response of turnout to discounting. We found a negative relation between discounting and turnout,

¹⁵Which is done modifying the probability of changing from state, ρ . Its modification introduces higher noise in the system and, higher changes in individuals income, that may be either positive or negative.

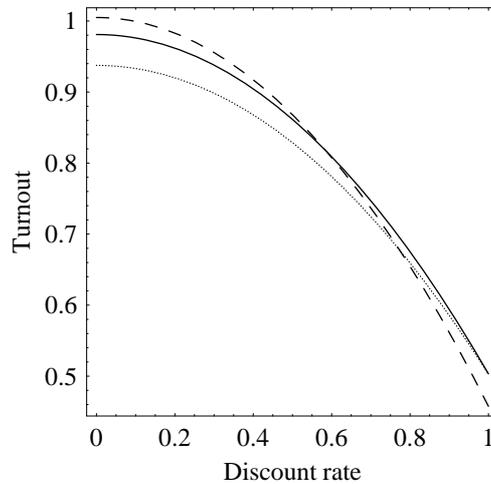


FIGURE 2. Effect of the discount rate on the same population, iterated for different ρ -values: (i) dashed line: $\rho = 0.25$; (ii) continuous line: $\rho = 0.5$; (iii) dotted line: $\rho = 0.75$.

as expected, and that a population tends to be more sensitive to discount rate the lower the ρ -parameter they face. Simulations were made with three different values: $\rho' = 0.25$, $\rho'' = 0.5$ and $\rho''' = 0.75$; the corresponding estimated coefficients of expression (6) were: $\alpha_1' = -0.54$, $\alpha_1'' = -0.47$ and $\alpha_1''' = -0.43$ respectively. These results were robust for different populations.

In the second simulation two alternative ρ -values were applied to 4179 different populations: in each case 21 different discount rates were used, ranging from 0 to 1 with a $\Delta\beta = 0.05$, leading to 199 populations generated for each discount parameter value. Figure 3 shows the effect of the discount rate on turnout for this set of populations. Here we present the average value for participation, given different populations. Again a set of least squares quadratic functions would arise in the case we estimated them, but it seems unnecessary as the main conclusion from the previous experiment arise again in this setting. Average turnout levels tend to be lower for higher discount rates; although it can be seen that high turnout levels are possible for high discount rates the dispersion of the points is broader in the latter. This accounts for the fact that it may be possible that high discount rates lead to relatively high turnout values, as initial conditions drive the future evolution of a given population: history matters. From the comparison of the top and bottom graphics, it is evident that the dispersion is larger for higher economic instability (lower ρ value), and that the top graph shows a steeper set of theoretical curves, which account for the same facts as in the one population case.

Finally we analyzed how individually perceived net benefits affect voting behavior. In a recent paper, Fleck (1999) shows how political participation may affect the distribution of government funds: in this work incumbents are more

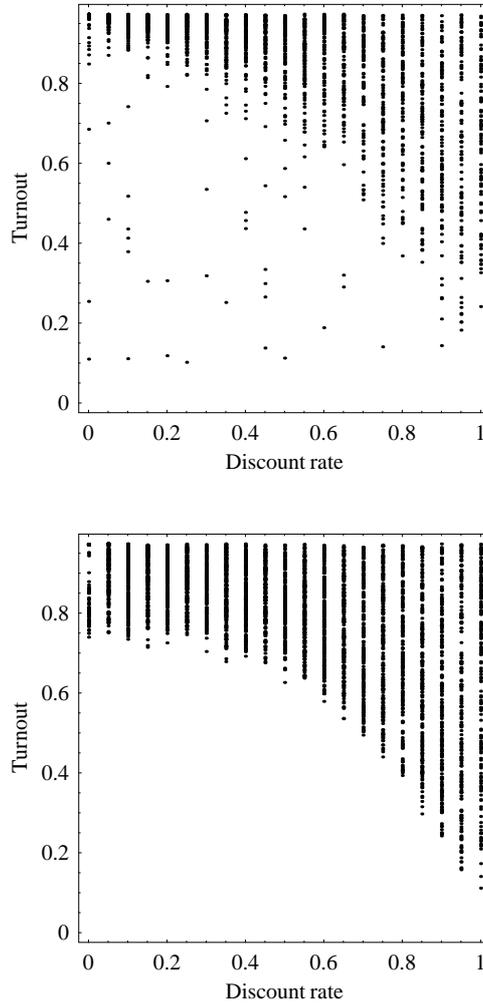


FIGURE 3. Effect of the discount rate for different populations and different ρ -values. Top: $\rho = 0.25$; bottom: $\rho = 0.75$.

likely to be elected if high-turnout regions receive more funds than low turnout regions, hence funds will be allocated in high turnout regions. We may consider in our setup that the redistributive policy can be managed in order to get further support; making the simplifying assumptions that each individual is a region and the winning party may distribute funds only to those voters who supported it in the past¹⁶. To this end we simulated the same population under two different redistributive regimes: in the first redistribution as per expression (1) applies (which we

¹⁶This is to consider that regions may receive more benefits if they supported the winning party. Of course this assumption is too extreme and used only for illustrative purposes.

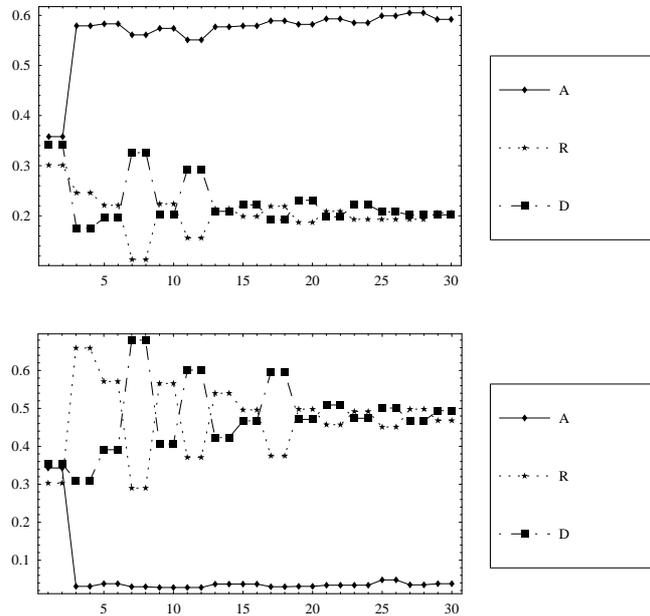


FIGURE 4. Different distributive politics programs: top, universal distribution; bottom, selective distribution. (A: abstentions).

will call universal distribution); in the second only individuals voting the winning party receive transfers from the government (we called this case selective distribution). Results, Figure 4, are supportive of the hypothesis. If individuals do not vote, they will be facing the opportunity cost of not receiving transfers, thus participation is significantly higher in the case of selective distribution. Voters act rationally by voting in this second case and the *Fleck hypothesis* holds without the need to resort to rationality assumptions about individual behavior.

3.2. ANALYSIS OF POLITICAL OUTCOMES

Now we are going to focus on the outcomes of political processes, that in our simplified model refers to the collective choice of a redistributive policy. Table 1 show the main results of modifying the relative rate of redistribution of the political parties for a given population. Two features arise: the average of votes of both parties was similar most of the time; and so was the fraction of times that both parties were winners although in most cases the R party (offering a lower level of redistribution) had an advantage. This was specially evident for a radicalized D party offering high levels of redistribution. However it should be noted that these results are not surprising as most of the time, and for a given redistributive policy there will be a significant share of the population that will benefit from it. Table 2, by contrast, shows the effect of modifying the economic stability, while keeping

TABLE 1. Analyzing alternative redistributive policies for D party. R redistribution is pegged to 10%. (FW: fraction of times the party won an election; AV: average number of votes).

D Policy	R Outcomes		D Outcomes		Abstention
	FW	AV	FW	AV	AV
11%	46%	445	54%	447	108
20%	56%	458	44%	456	85
30%	37%	445	63%	451	105
50%	67%	415	33%	410	362
70%	53%	319	47%	318	362
90%	67%	399	33%	394	207

TABLE 2. Analyzing alternative economic settings (R and D policies are fixed to 10% and 60% respectively). (FW: fraction of times the party won an election; AV: average number of votes).

ρ	R Outcomes		D Outcomes		Abstention
	FW	AV	FW	AV	AV
0.00	53%	421	47%	420	158
0.25	50%	324	50%	324	352
0.50	47%	402	53%	405	193
0.75	51%	111	48%	111	778
1.00	48%	327	51%	328	345

both policies constant. Although results coincide with our previous comment, it should be noted that the fraction of times the D party won an election tends to be slightly higher for higher economic instability.

4. CONCLUSIONS

The previous pages showed the results on the simulation of a very simplified experimental political economy. This way of modeling differs in a significant way from orthodox economic theory, where rational individuals interact maximizing some well defined objective function. However if this approach is of limited scope in economics, it has more drawbacks in the political exchange where the means and objectives of individuals are fuzzily defined. Moreover, our approach benefits from the fact that individuals do not possess an invariable view of the real world but it changes over time as they learn from situations they experienced and revise their previous views. In this sense the adaptive, or non rationalist, alternative to explain collective action proposed by Suzuki (1991) may well be based on models like the one developed here, in which boundedly rational individuals interact in the political market. Furthermore it shall be noted that different types of behavioral programs arise from this setting by modifying certain structural parameters.

For instance, setting the discount rate to zero would imply an adaptive (backward looking) model of political behavior, in which the past determines the outcomes in the future. But it seems plausible that individuals, although myopic, may also take into consideration future events.

Behavioral theories raise interesting questions about rationality. Here the rationality of any program is to be analyzed in term of the alternatives individuals are facing: from this point of view, a player would be said to act rationally if, given the set of alternative options she is facing, she chooses the one which will yield the highest payoffs. As agents endowed with a classifier system are choosing the fittest rules at every stage of the interaction, the behavioral program will be driving agents rationally.

Finally computational experiments addressed to contrast our model results with those of the empirical and theoretical literature. Analyzing the outcomes of the simulations it has been shown that the agents do not depart from the degree of rationality that it is assumed in political economy models of voting. In fact most of the hypothesis postulated in the literature have been show to hold in our model: constituency size effects, expected benefits and expectations about the future play a role in determining the degree of participation in an election. In addition, computational results reconcile theory and facts. And this without resorting to specific irrational behavior or getting paradoxical outcomes.

REFERENCES

- Alesina A. and Rosenthal H. (1995) *Partisan politics, divided government and the economy*. Cambridge university press, Cambridge.
- Başçı E. (1999) Learning by imitation. *J. Econ. Dynamics and Control* **23**, 1569–1585.
- Black D. (1948) On the rationale of group decision-making. *J. Political Economy* **56**, 23–34.
- Buchanan J.M. (1954) Individual choice in voting and the market. *J. Political Economy* **62**, 334–343.
- Buchanan J.M. and Tullock G. (1962) *The calculus of consent*. The University of Michigan Press.
- Campbell C.M. (1999) Large electorates and decisive majorities. *J. Political Economy* **107**.
- Conlisk J. (1996) Why bounded rationality? *J. Econ. Literature* **34**.
- Downs A. (1957) *An Economic Theory of Democracy*. Harper & Row, New York.
- Fleck R.K. (1999) The value of the vote: a model and test of the effects of turnout on distributive policy. *Economic Inquiry* **37**.
- Hibbs D. (1977) Political parties and macroeconomic policies. *Am. Political Sci. Rev.* **71**, 1467–1487.
- Holland J.H. (1992) *Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence*. MIT Press, Cambridge, MA.
- Ledyard J. (1984) The pure theory of large two candidate elections. *Public Choice* **44**, 7–41.
- Lettau M. & Uhlig H. (1999) Rules of thumb versus dynamic programming. *The Am. Econ. Rev.* **89**, 148–174.
- Levine D. *PGAPack V 1.0 library for Linux*. <http://www-unix.mcs.anl.gov/~levine/PGAPACK/>.
- Mailath G.J. (1998) Do people play Nash equilibrium? Lessons from evolutionary game theory. *J. Econ. Lit.* **36**(3), 1347–1374, September.

- Marimon R., McGrattan E. and Sargent T. (1990) Money as a medium of exchange in an economy with artificially intelligent agents. *J. Econ. Dyn. Cont.* **14**, 329–373.
- Miller G.J. (1997) The impact of economics on contemporary political science. *J. Econ. Lit.* **35**, 1173–1204.
- Mueller D.C. (2000) Capitalism, democracy and rational individual behavior. *J. Evol. Econ.* **10**, 67–82.
- Nelson R. (1995) Recent evolutionary theorizing about economic change. *J. Econ. Lit.* **33**, 48–90.
- North D.C. (1993) What do we mean by rationality? *Public Choice* **77**, 159–162.
- Palfrey T. and Rosenthal H. (1984) Participation and the provision of discrete public good: a strategic analysis. *J. Public Economics* **24**, 171–193.
- Palfrey T. and Rosenthal H. (1985) Voter participation and strategic uncertainty. *Am. Political Sci. Rev.* **79**, 62–78.
- Palfrey T. and Rosenthal H. (1987) The downsian model of electoral participation: formal theory and electoral analysis of the constituency size effect. *Public Choice* **52**, 15–33.
- Peltzman S. (1990) How efficient is the voting market? *J. Law and Econ.* **33**, 27–63.
- Piketty T. (2000) Voting as communicating. *Rev. Econ. Studies* **67**, 169–191.
- Ryker W. and Odeshook P.C. (1968) A theory of the calculus of voting. *Am. J. Pol. Econ.* **62**, 25–42.
- Shachar R. and Nalebuff B. (1999) Follow the leader: theory and evidence on political participation. *Am. Econ. Rev.* **89**, 525–547.
- Suzuki M. (1991) The rationality of economic voting and the macroeconomic regime. *Am. J. Political Sci.* **35**, 624–642.
- Swank O. (1998) Partisan policies, macroeconomic performance and political support. *J. Macroeconomics* **20**, 367–386.
- Vriend N.J. (2000) An illustration of the essential difference between individual and social learning and its consequences for computational analysis. *J. Econ. Dyn. Cont.* **24**.