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A MODEL OF MARKET-MAKING

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Abstract. The two essential features of a decentralized economy taken into account are, first, that individual agents need some information about other agents in order to meet potential trading partners, which requires some communication or interaction between these agents, and second, that in general agents will face trading uncertainty. We consider trade in a homogeneous commodity. Firms decide upon their effective supplies, and may create their own markets by sending information signals communicating their willingness to sell. Meeting of potential trading partners is arranged in the form of shopping by consumers. The questions to be considered are: How do firms compete in such markets? And what are the properties of an equilibrium? We establish existence conditions for a symmetric Nash equilibrium in the firms' strategies, and analyze its characteristics. The developed framework appears to lend itself well to study many typical phenomena of decentralized economies, such as the emergence of central markets, the role of middlemen, and price-making.

Classification Codes. C7, D8, L1, M3.

1. Introduction

Consider the following quote:

> *"Markets rarely emerge in a vacuum, and potential traders soon discover that they may spend more time, energy, and other resources discovering or "making" a market than on the trade itself. This predicament is shared equally by currency traders, do-it-yourself realtors, and*

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streetwalkers! Their dilemma, however, seems to have gone largely unnoticed by economists, who simply assume that somehow traders will eventually be apprised of each other's existence - to their mutual benefit or subsequent regret".(Blin, 1980, p. S193)

Since this quote perfectly captures the motivation behind our paper, and the essence of what this paper is all about, we move directly to the substance of our paper. In order to deal with the indicated lacuna, we will analyze a model of competition with market-making for a decentralized economy characterized by the following basic properties: (*i*) There is a large number of agents and a large number of commodities. The agents and their physical environment are characterized by preferences, endowments, and technologies. (*ii*) Each agent is interested in only a limited number of commodities, while the fraction of agents interested in a given commodity is small for each commodity (*cf.* Fisher, 1983). (*iii*) The economy is not organized by an auctioneer, intermediary, specialized trader, central distributor, or anonymous random matching mechanism (*e.g.*, Gale, 1985), but is instead one with decentralized trade that depends upon the decisions of individual agents who act on a strictly do-it-yourself basis. (*iv*) Agents, although knowing about the existence of other agents, have no further pre-communication knowledge about each other, such as, for example, their effective demands; not to mention characteristics as endowments and preferences. (*v*) An agent who does not possess any information about the characteristics of other agents is not in a position to find a trading partner. (*vi*) Individual agents may communicate with each other. (*vii*) Communication is costly. (viii) There is no exogenous, *'state-of-nature'* uncertainty. (*viii*) Individual agents know the aggregate state of the economy, *e.g.*, aggregate demands, total numbers of sellers/buyers. (*ix*) All commodities are known by all agents (*cf.* Gary-Bobo & Lesne 1988), and there is no quality uncertainty (*cf.* Spence, 1974). (*x*) All transaction costs are information costs, and there are no real transaction costs (see Shubik, 1975).

The assumed properties imply two essential features of a decentralized economy. First, the market interactions that take place depend in an essential way upon the knowledge of the identity of some other agents in the economy[2]. The key property here is (*v*), which may be explained by the fact that the perceived costs of uninformed search are greater than its perceived gains. This may be due to the fact that the *'psychological'* costs (disutility) of accosting a randomly chosen, unidentified agent to bother him with a question like *'Could you please sell me a refrigerator?'* are high, while the probability that such an agent will in fact be interested in such a transaction is low[3]. Through communication with other agents, an individual agent creates the possibility of meeting potential trading partners. In general, when there are possibilities of trading in a certain commodity, it is said

[2]See also Stigler (1961), who points out that the issue of the identity of sellers logically precedes the information problem concerning prices.

[3]Although this is related to the other properties, it is convenient to assume it directly. To give one example of the costs (disutility) of uninformed search: A smoker, after having troubled in vain two non-smokers for a light, might hesitate before asking the next passer-by, and will probably wait until he sees someone actually smoking.

that a market for that commodity exists. Hence, by establishing communication with other agents, individual agents create their own markets. In other words, a market is not a central place where a certain good is exchanged, nor is it simply the aggregate supply and demand of a good. A market is constituted by communication between individual agents. As we will show in this paper, the second essential feature implied by the above assumed properties is that in general the agents will face trading uncertainty[4]. Notice that in the sketched non-Walrasian trading structure[5] individually relevant information would be complete only when each agent knows not only the current price vector, but also the complete vector of effective demands of all other agents, and where and when to meet these agents.

In Section 2 we will make these general assumptions concrete, and discuss the specific questions to be considered. In Section 3 we will derive an equilibrium market structure from the optimizing behavior of individual agents, showing necessary and sufficient conditions for its existence, and analyze its characteristics, while Section 4 will conclude.

2. The model

Agents and commodities

Assuming time to be divided into an infinite sequence of discrete periods indexed τ, $\tau \in \{1, 2, ...\}$, we will consider an economy with a single homogeneous, perishable commodity. A set A of N agents, each characterized by preferences, technologies and endowments, is divided into two disjoint classes: a set B of firms and a set D of consumers, with $|B| = m$, $|D| = N - m$, $B \cap D = \varnothing$, and $B \cup D = A$. We think of N as *'large'*. Given the agents' preferences, technologies and endowments, any given consumer i can be characterized by a threshold price $\bar{p_i}$. This threshold price $\bar{p_i}$ corresponds to the utility U_i the agent would derive from the consumption of one unit of the commodity, and is the price above which this agent would certainly not purchase a unit of the commodity (see *e.g.*, Gale, 1985 or Kormendi, 1979). Formally $\bar{p_i}$ is defined by $U_i(0, \omega_i) = U_i(1, \omega_i - \bar{p_i})$, where the first argument of the utility function concerns the commodity considered, and the second represents a *'basket'* of other goods or *'income'*. We assume that the threshold price is 0 for all consumers with respect to any additional unit of the commodity. Formally: $U_i(1, \omega_i - \bar{p_i}) = U_i(a, \omega_i - \bar{p_i}) \, \forall \, a \geq 1 \, \forall i$. Thus, the number n of interested consumers depends upon the price p, and the aggregate demand may be written $n(p)$, which we assume to be objectively known by all sellers. In other words, given the characteristics of the individual agents, for each price p the set D of consumers consists of $n(p)$ potential buyers and $N - m - n(p)$ agents who

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[4]Hahn (1980) stresses the distinction between trading uncertainty, which means being uncertain as to whether or not an agent will be able to trade what he wants at the going price, and price uncertainty; soliciting an analysis of the first. In productions and operations analysis this is known as the *'newsboy problem'* (see, *e.g.*, Nahmias, 1993).

[5]See Vriend (1994) for an extensive discussion of Walrasian trading structures.

are totally uninterested in the commodity. The m firms produce and sell the commodity. They are assumed to be identical in that they use the same technology. The cost C of producing z units of output is given by the function $C(z)$, where $z \in \mathbb{N}$. We assume $C(0) \geq 0$ and $\mathrm{d}C(z)/\mathrm{d}z > 0 \ \forall z \geq 0$[6]. The production decided upon at the beginning of the period is immediately available for sale, while unsold stocks perish at the end. In order for trade to take place, firms and consumers must meet, and they must agree upon the terms of trade. We assume that the price p of the commodity is given and equal for all firms and consumers, and that it is known to all agents. We assume that agents know that other agents exist, but they do not know any of the characteristics of these agents. In particular, they do not know which agent belongs to which class. As some information in this respect is an essential pre-requisite for meeting potential trading partners, we have to specify the way in which agents communicate with each other, *i.e.*, how they create their own markets.

Communication and trade

Each seller may send information signals to some other agents at the beginning of the period, each signal being directed to one agent. A signal contains, first, the *'name and address'* of the sending agent, and second, the fact that he belongs to the class of firms B. Thus, a signal reveals the type of a given agent. Agents who neither perceive nor send signals cannot find a trading partner. We assume that signaling is costly, the cost being an increasing function of the amount of signals sent. The cost K of sending s signals is given by the function $K(s) = k \cdot s$, where $s \in \mathbb{N}$ and $k > 0$. Thus, we assume constant marginal signaling costs, $\mathrm{d}K(s)/\mathrm{d}s = k$, while $K(0) = 0$. Receiving signals, on the other hand, is costless.

Agents make their decisions concerning communication and effective demand at the beginning of each period. During each period they try to buy or sell in their markets. We assume that the trading possibilities for each agent are dependent only upon the communication and demands in the given period. Thus, sellers have no reputation, and there are no customer relations. Moreover, the demand above the firm's available supply is simply foregone, and cannot be backlogged. We assume that each consumer who has received one or more signals may visit one firm (*'shopping'*). The order in which buyers make their visit is random[7]. When

[6] For reasons of expositional convenience, the notation used in this paper will usually obscure the fact that some variables, in particular z and s, are discrete. We will write, for example, $\mathrm{d}g(x, y, ...)/\mathrm{d}x$ instead of $g(x+1, y, ...) - g(x, y, ...)$ for any function $g(x, y, ..)$.

[7] To assume that consumers may make only one visit and that the order in which they make their visit is random, is only for convenience. It does not restrict the nature of the problem of the firm; it simply changes the value of some of its parameters. It is a simplified version of the more general scenario in which consumers can make more visits during each period, also on more markets to buy various commodities, while the shopping behavior is not synchronized between the consumers. Let us assume that time flows continuously, that the visits and exchanges are discrete events of zero duration, like the arrivals in a Poisson process (see Foley, 1975 or Diamond, 1982), associate with each agent a *'random clock'* that rings independently for each agent at the instances of a Poisson process, and let each consumer make a visit when his clock rings (see, *e.g.*,

a firm is sold out, customers will return home dissatisfied; if not, they may buy one unit[8].

Discussion of the model

The model is set up in order to focus on the role of communication about the identity of agents, and to analyze the resulting trading uncertainty. We believe the most convincing arguments to follow this approach are *empirical*. First, communication to identify potential trading partners is important in real markets. Some readers may wonder why we bother to develop this model with its explicit signaling structure, since in the real world *'a consumer simply knows where to buy something'*? Well-considered, such an observation gives strong empirical support for our approach. For it asserts that transactions do not take place in Walrasian central markets, or through anonymous random matching devices, but that, instead, market interactions depend in a crucial way on local knowledge of the identity of potential trading partners. Such information has to be communicated in one way or another. Most advertising in reality seems indeed to draw the buyers' attention to the fact that someone is selling something somewhere sometime. Notice that the advertising signals do contain no information on prices, and that this conforms to what we usually observe in reality. Although Stigler (1961) argued: *"From the manufacturer's viewpoint, uncertainty concerning his price is clearly disadvantageous. The cost of search is a cost of purchase, and consumption will therefore be smaller"* (p. 223), daily experience suggests that sellers must perceive some advantage of *not* communicating price information[9]. Advertising is not always in the form of signals sent directly to individual potential buyers. Agents may also hear from radio and tv, or from friends about the newest shops in town, buyers may use the Yellow Pages to find a seller, or they may visit shops (recognizable as such) randomly, etc. While these possibilities would require slight modifications of the signaling framework used, they would seem to fit rather well into it. The essential characteristic of all these forms of signaling is that agents make information about their own type known to some other agents. And as argued already in the opening quote by Blin, the resources spent in these forms of market-making are enormous in a modern market economy.

Griffeath, 1979). If the length of a period τ is finite, each consumer will be able to make only a limited number of visits in each period

[8]Thus, trade in our model is bilateral. In this sense we differ from Ioannides (1990), who also analyzes communication by individual agents in order to make markets, and then considers multilateral trade between all agents who are directly or indirectly informationally linked.

[9]Not only do sellers usually advertise without communicating prices, they often furnish shop-windows, and sometimes even have shelves, without prices. A buyer who incurs costs to get such information, *e.g.*, by making a phone call, can often expect a rude answer or sometimes no answer at all, and a buyer who writes down prices in a supermarket takes some risk of being threatened. And even when advertising signals appear to contain information on prices, this is not necessarily meant as informative as it appears to be. Lazear (1995) analyzes the practice of using price signals in advertising as a bait to attract customers, in order to switch to different products with different prices instead when customers visit their shop.

Second, markets with trading uncertainty are empirically relevant. There *are* many markets in which goods are sold at fixed prices (whether as a result of legislation, of vertically imposed restrictive practices, or of optimizing behavior of the sellers), and over the period for which prices are fixed, trading opportunities are usually uncertain. Doctors fees are regulated in many countries, the prices of books are fixed by publishers' cartels in a lot of countries, bread prices in Italy are determined centrally, newspapers usually sell for the same constant price at all stands, etc. Nearly any retail operation has a posted price, invariant through time, orders for stock are placed in advance of knowing what demand will be, and stock-outs are commonly faced. Levis jeans are sold that way. So are Seiko watches. McDonald's rarely runs out of food, but it does happen. A fascinating market where this is true is the motion picture industry (see De Vany & Walls, 1996). Admission price is set and does not vary over the run, films are booked at theaters before demand is known, customers cannot always get a seat at their preferred showing and must queue up or wait for another day. Also custom items, *e.g.*, in the finished steel or medical industry, usually are produced to order, at previously posted prices[10]. Clearly, a *complete* economic analysis would explain such legislation, restrictive practices, or strategies, by which the prices are fixed, as well[11]. But that is not the aim of this paper. Instead of explaining the posted prices, our analysis focuses on the direct consequences for the competitive process within the industry itself; and thus applies equally to all the possible ways in which these prices may have been determined[12]. Given prices, competition can take place along many dimensions. As argued above, competing for customers by making its identity known through signals is a fundamental one. Therefore, focusing on the issue concerning the identity of traders, and the resulting trading uncertainty, the first questions to be considered are: How do firms compete in such markets? Does an equilibrium exist? And what are the properties of an equilibrium?

[10]The list of examples seems to suggest that this applies to all markets that are not centralized like stock markets or auctions. But notice that even the Israeli stock market collects bids and offers to find a clearing price only in the morning, and then that price is posted and remains *fixed* over the entire trading day. On the other hand, there are also examples of markets with decentralized trade where prices are not posted; for example the Fulton fish market (see Graddy, 1995), or the Marseille fish market (see Kirman & Vriend, 2001). Further documentation concerning the (in)flexibility of prices can be found in Carlton (1989).

[11]For example, Riley & Zeckhauser (1983), who consider the question whether to haggle or not when buyers come in with bids in sequence, show that in a wide variety of markets a fixed, posted price is an optimal strategy for a seller. See also Ross (1993), who finds a similar property solving the so-called streetwalker problem.

[12]This is also in the spirit of the temporary equilibrium literature (see Grandmont, 1988a for an overview), in which prices are supposed to have been quoted at the outset of the period, as the result of an *unspecified* process of imperfect competition, and to remain momentarily fixed during the period analyzed. Some other papers following this basic approach are Carlton (1978), De Vany & Saving (1983), and Gould (1978).

3. Strategies and equilibrium

Objectives and strategies

We first consider the agents' objectives and strategies. A consumer's utility would increase if he could buy one unit of the commodity at a price below his threshold price $\bar{p}$. If a consumer receives one or more signals at the beginning of the period, and if the price is indeed below his threshold price $\bar{p}$, then he chooses among these signals a firm to visit. Since all firms have the same price, and the signals as such convey no further information about the firms' service reliability, we can think of the consumers picking a firm at random. If this firm is sold out, the consumer will be left dissatisfied in this period, otherwise he will buy one unit.

Firm i's objective is to maximize its expected current profit V_i, which is equal to its expected gross revenue R_i minus its production cost $C(z_i)$ minus its market-making cost $K(s_i)$, by deciding upon its effective supply z_i and signaling s_i. Moreover, it has to decide to which agents it will send these signals. We assume that the destination of each signal is chosen at random. Thus a strategy t_i of firm i is a pair $(z_i,\ s_i)$. The costs are dependent only upon firm i's own strategy, but the revenue of its market-making and production activity, *i.e.*, firm i's actual sales ($p \cdot x_i$, to be specified below), are a function of the vector $\mathbf{t}$ of the strategies of all firms (including firm i itself)[13]. Thus, firm i's objective function is: $V_i(\mathbf{t}) = R_i(\mathbf{t}) - C(z_i) - K(s_i)$, where $R_i(\mathbf{t}) = p \cdot E(x_i(\mathbf{t}))$.

In this model of a decentralized economy, in which the firms act simultaneously, we restrict our attention to the existence and characterization of Symmetric Nash Equilibria (SNE) in the firms' pure strategies. We neglect the possibility of asymmetric equilibria as they should not be expected in one- population models. It might be that a preplay communication phase could enable the agents to coordinate on some asymmetric equilibrium, but the main idea of this paper is to make the communication structure explicit. Hence, implicitly assuming that there does exist a communication structure through which the actions of the agents might be coordinated in order to reach an equilibrium, would seem paradoxical and counter-productive.

Stochastic trading opportunities

Firm i's gross revenue is equal to its sales, $p \cdot xi(\mathbf{t})$. Given the assumptions made in the previous sections about the nature of the commodity and the trading structure, it is clear that firm i cannot sell more than is demanded by its customers, $q_i(\mathbf{s})$, or than it has produced at the beginning of the period, z_i. That is, $x_i(\mathbf{t}) = \min\{q_i(\mathbf{s}), z_i\}$. Therefore, we must specify the demand directed towards firm i, $q_i(\mathbf{s})$.

Proposition 1. *The demand directed towards firm i, $q_i(\mathbf{s})$, is a random variable given by a Poisson distribution with parameter $\mu_i = \mu(s_i, S_{-i}) = (s_i/S) \cdot n(p) \cdot$*

[13] Vectors are denoted by **bold-face** letters.

$(1 - \mathrm{e}^{-S/N})$*, where* S_{-i} *denotes the aggregate number of signals sent by the other firms, and* $S = s_i + S_{-i}$.

Proof. See Vriend (1996). Here we give an outline only[14]. Firm i sends s_i signals at random into the population. A given signal sent has success when it induces a consumer wanting to buy one unit to visit firm i. This depends upon the probability that the receiver of such a signal is an interested consumer and the probability that he will choose the signal from firm i among the signals he receives. The latter, clearly, also depends upon the aggregate signaling activity of the other firms. It turns out that the probability that any given signal sent by firm i will lead to a consumer visiting firm i is given by $\Pr(S) = (n(p)/S) \cdot (1 - \mathrm{e}^{-S/N})$. Firm i sends s_i signals, and the number of buyers visiting firm i may be approximated by a Poisson distribution with parameter $\mu(s_i, S_{-i}) = s_i \cdot Pr(S) = (si/S) \cdot n(p) \cdot (1 - \mathrm{e}^{-S/N})$. □

This result can be read as follows. The potential aggregate demand in the economy, given the price p, is n[15]. The probability that a given potential consumer will not receive any signal at all, and thus will not find his way to a market, is $\mathrm{e}^{-S/N}$. Hence, aggregate market demand is $n \cdot (1 - \mathrm{e}^{-S/N})$. Finally, each firm's expected market share turns out to be equal to his share in the aggregate market-making activity, s_i/S. Note that the probability of any single signal from firm i having success is a function only of the aggregate number of signals S sent by all firms. Hence, $\Pr_i(S) = \Pr(S)\, \forall i$. This is because each interested consumer handles all his received signals identically, putting them all in an urn and drawing just one signal. Notice also that μ_i turns out to be a function only of the number of signals sent by firm i itself, s_i, and the aggregate number of signals sent by all other firms, S_{-i}. Thus, the vector of strategies chosen by the other firms $\mathbf{t_{-i}}$ enters firm i's decision problem only through the aggregate market-making signaling activity[16]. The resulting transaction possibilities for any given agent are stochastic. Thus agents are uncertain as to whether they will be able to trade as much as they want. There are, as we have seen, two direct causes for this trading uncertainty. First, communication is stochastic, *i.e.*, signals are randomly distributed as agents do not know each other's characteristics. Second, given that an agent has found or established a market, either he or his potential trading partners may have already fulfilled their demand before they happen to meet, *i.e.*, shopping is a stochastic process. Note that the trading possibilities for firm i are derived explicitly from assumptions with regard to the underlying communication and trading structure of the economy, instead of assuming directly a functional form of each agent's trading possibilities. The stochastic demand for firm i's output depends upon one of the (non-price) decision variables of the firm itself. This stochastic demand is not generated by sending an effective demand (*i.e.*, supply) to the market, but by

[14] For many propositions and claims we will not present the sometimes cumbersome complete proofs. Where essential, an outline of the proof, plus the underlying intuition will be presented. The complete proofs can be found in Vriend (1996).

[15] To lighten notational burden somewhat, we will usually write n instead of $n(p)$.

[16] Allowing for more visits per buyer would simply give a higher value for the parameter μ.

creating the market itself[17]. As a result, firm i's expected gross revenue may be written as follows, where $f[|\mu]$ denotes the p.d.f. with parameter μ:

$$R_i(z_i, s_i, S_{-i}) = p \cdot \sum_{qi=0}^{z_i} qi \cdot f[q_i|(s_i, S_{-i})] + z_i \sum_{q_i=z_i}^{\infty} f[q_i|\,\mu(s_i, S_{-i})]. \qquad (1)$$

Observe that the stochastic trading mechanism has an anonymity property. That is, agents who have the same effective demand and have sent out the same number of signals can expect the same realizations. This is due to the fact that trading possibilities depend only upon current period variables, that all signals are for each firm independently distributed, each agent being equally likely to receive such signals, that the firms to visit are chosen independently by all buyers, each firm being equally likely to be chosen among the firms in the buyer's market, and that the order in which buyers make their visits is determined randomly and does not depend upon the agents themselves. We further characterize the stochastic demand directed to firm i through the following claims.

Claim 1.

a: For given S_{-i} , $\mu(s_i, S_{-i})$ is a one-to-one function of s_i , which satisfies: $\mu(0, S_{-i}) = 0$, $0 \leq \mathrm{d}\mu(s_i, S_{-i})/\mathrm{d}s_i \approx Pr(S) \leq 1, \mathrm{d}^2\mu(s_i, S_{-i})/\mathrm{d}s_1^2 < 0$, and $\lim_{s_i \to \infty} \mu(s_i, S_{-i}) = n$.

b: For given s_{-i} , $\mu(s_i, S_{-i})$ is a one-to-one function of S_{-i} , which satisfies: $\mathrm{d}\mu(s_i, S_{-i})/\mathrm{d}S_{-i} < 0$, and $\lim_{s_{-i} \to \infty} \mu(s_i, S_{-i}) = 0$.

Proof. See Vriend (1996). □

Thus, a firm that does not signal does not get any demand. The expected change in the demand directed to firm i as a result of sending one additional signal is positive but less than 1, and it depends only upon the aggregate signaling activity in the economy. Notice that for given S this is equal for all firms, and that it is not important which firms send these signals, and in particular it does not matter how many of the S signals are sent by firm i itself. A firm may eventually capture the whole aggregate demand by signaling more and more, given the strategies of the other firms. However, the more the other firms signal, the less will firm i's expected demand be. Suppose all m firms send the same number of signals: $s_i = s\,\forall i$, $S = m \cdot s$, and $\mu(s_i, S_{-i})$ becomes $\grave{\mu}(s) = \mu(s, (m-1) \cdot s)$.

Claim 2. $\grave{\mu}$(s) is a one-to-one function of s, which satisfies: $\grave{\mu}(0) = 0$, $0 \leq \mathrm{d}\grave{\mu}(s)/\mathrm{d}s \leq 1$, $\mathrm{d}^2\grave{\mu}(s)/\mathrm{d}s^2 < 0$, and $\lim_{S \to \infty} \grave{\mu}(s) = n/m$.

[17] In this sense we differ from the literature on stochastic rationing (*e.g.*, Green, 1980), where it is directly assumed that each agent's trading possibilities are a stochastic function only of his own demand and the aggregate demand and supply in the economy. We also differ from the literature on completely random matching models (*e.g.*, Gale, 1985), where an agent's trading opportunities are independent from his own decisions. And we differ from the fixed price literature in general, where the sending of effective demands is the only means of communication (*cf.*, Drazen's, 1980 criticism).

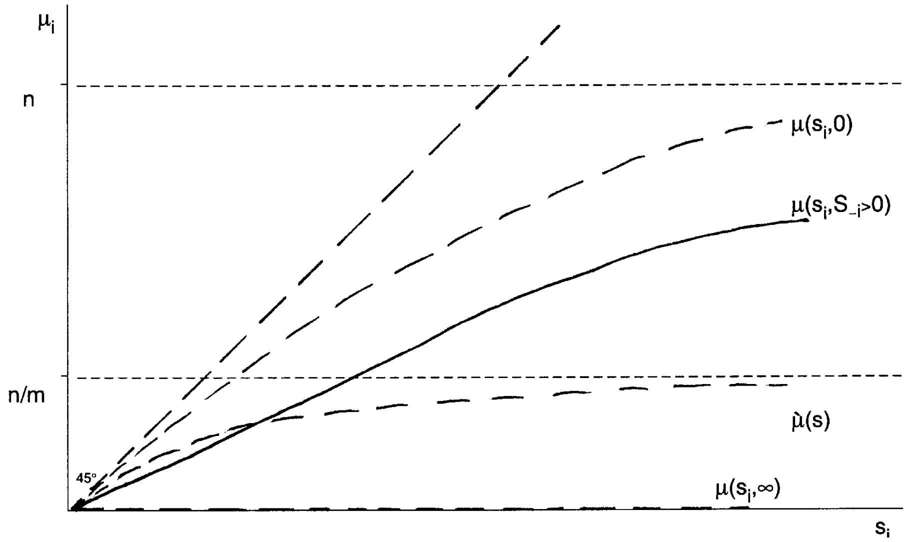

FIGURE 1. Expected demand.

Proof. See Vriend (1996). □

Thus, if all firms send an infinite number of signals they may expect to share equally the whole aggregate demand. Note that $m = 1$ corresponds to the case of a monopolist. Claims 1 and 2 are illustrated in Figure 1.

Optimization and equilibrium

We are now in a position to consider firm i's optimization problem. As a strategy t_i for firm i is a pair $(z_i,\, s_i)$, the first-order conditions (FOCs) for maximization of firm i's payoff are a system of two equations[18]:

$$\begin{aligned} \mathrm{d}V_i(z_i, s_i, S_{-i})/\mathrm{d}s_i &= \mathrm{d}R_i(z_i, s_i, S_{-i})/\mathrm{d}s_i - \mathrm{d}K(s_i)/\mathrm{d}s_i = 0 \\ \mathrm{d}V_i(z_i, s_i, S_{-i})/\mathrm{d}z_i &= \mathrm{d}R_i(z_i, s_i, S_{-i})/\mathrm{d}z_i - \mathrm{d}C(z_i)/\mathrm{d}z_i = 0. \end{aligned} \tag{2}$$

Claim 3.

a: $\mathrm{d}R_i/\mathrm{d}s_i = p \cdot F[z_i - 1] \cdot Pr(S)$, where $F[z]$ denotes $\sum_{q=0}^{z} f[q]$.
b: $\mathrm{d}R_i/\mathrm{d}z_i = p \cdot (1 - F[z_i])$.

Proof. See Vriend (1996). □

In other words, the gross revenue for firm i of sending one additional signal, given the strategies of the other firms, is the price p multiplied by the probability

[18] Remember that, in fact, the variables z and s are discrete. Hence, considering unit increments of these variables, the *'true'* FOCs are
$V_i(z_i, s_i, S_{-i}) - V_i(z_i - 1, s_i, S_{-i}) > 0$ while $V_i(z_i + 1, s_i, S_{-i}) - V_i(z_i, s_i, S_{-i}) \leq 0$
$V_i(z_i, s_i, S_{-i}) - V_i(z_i, s_i - 1, S_{-i}) > 0$ while $V_i(z_i, s_i + 1, S_{-i}) - V_i(z_i, s_i, S_{-i}) \leq 0$

that firm i would have had still at least one unit of the commodity available, multiplied by the probability that this additional signal will lead to a consumer visiting firm i. And the gross revenue for firm i of supplying one additional unit of the commodity, given the strategies of the other firms, is the price p multiplied by the probability that it would have sold out otherwise. It is advantageous for firm i to increase its signaling s_i with one unit, as long as $\mathrm{d}R_i/\mathrm{d}s_i > \mathrm{d}K/\mathrm{d}s_i = k$. Similarly, it is advantageous for firm i to increase its supply z_i with one unit, as long as $\mathrm{d}R_i/\mathrm{d}z_i > \mathrm{d}C/\mathrm{d}z_i$. As we consider a SNE, having derived the FOCs for maximization of firm i's payoff, we evaluate these conditions only for those cases in which each firm chooses the same strategy. Hence, $z_i = z$ and $s_i = s\,\forall i$, $S = m \cdot s$, and by FOC$^+$ we denote a first-order-plus-symmetry condition.

Claim 4.

a: For every value of z there exists exactly one value of s, denoted by $s(z)$, for which the first FOC$^+$ is satisfied. This function is characterized by $s(0) = 0$, $\mathrm{d}s(z)/\mathrm{d}z \geq 0$, $\lim_{z\to\infty} s(z) = s^{\max} = \{s\colon \mathrm{d}R_i(z = \infty, s)/\mathrm{d}s_i = k\}$, and $s(z) \geq z\,\forall z$ as long as $s(z) < s^{\max}$. Moreover, $s^{\max} > 0$ if and only if $n/N > k/p$.

b: For every value of s there exists exactly one value of z, denoted by $z(s)$, for which the second FOC$^+$ is satisfied. This function is characterized by $z(0) = 0$, $\mathrm{d}z(s)/\mathrm{d}s \geq 0$, $\lim_{s\to\infty} z(s) = z^{\max} = \{z\colon \mathrm{d}R_i(z, s = \infty)/\mathrm{d}z = \mathrm{d}C/\mathrm{d}z\}$, and $z(s) \leq s\,\forall s$. Moreover, if $\mathrm{d}^2C/\mathrm{d}z^2 \geq 0\,\forall z$ then $z^{\max} > 0$ if and only if $n/m > -\ln(1 - \mathrm{d}C(0)/\mathrm{d}z/p)$[19].

Proof. See Vriend (1996). □

Both curves are drawn in Figure 2. Clearly, if a firm does not produce, it does not signal either, and vice versa. Moreover, there is a maximum level of signaling, which is related to the fact that beyond that level it is very unlikely that the receiver of an additional signal will respond to that signal. Thus, whatever the level of production the expected gains from an additional signal are below its costs. Similarly, there is a maximum level of production, which is related to the fact that it is very improbable that a customer will ever come to buy it, whatever the level of signaling. At a point of intersection of the two curves, both FOCs for maximization of firm i's payoff are fulfilled, while each firm chooses the same strategy $\hat{t}$.

Now, we turn to the second-order condition (SOC) for $\hat{t}$ to be a SNE strategy: $\mathrm{d}^2V_i(z_i, s_i, S_{-i})/\mathrm{d}z_i^2 \cdot \mathrm{d}^2V_i(z_i, s_i, S_{-i})/\mathrm{d}s_i^2 - (\mathrm{d}\{\mathrm{d}V_i(z_i, s_i, S_{-i})/\mathrm{d}s_i\}/\mathrm{d}z_i)^2 > 0$ and $\mathrm{d}^2V_i(z_i, s_i, S_{-i})/\mathrm{d}s_i^2 < 0$. The following claim gives a necessary and sufficient condition for this to be satisfied, given a strategy $\hat{t}$ at which the FOC+s are fulfilled.

Claim 5. $\{\mathrm{d}^2C(\hat{z})/\mathrm{d}\hat{z}^2\}/p > f[\hat{z} + 1]/\hat{z} \Leftrightarrow$ the SOC is fulfilled.

Proof. See Vriend (1996). □

[19] Anticipating a result of the analysis, we here avoid giving a rather cumbersome analogous expression for the case in which $\mathrm{d}^2C(z)/\mathrm{d}z^2 < 0$.

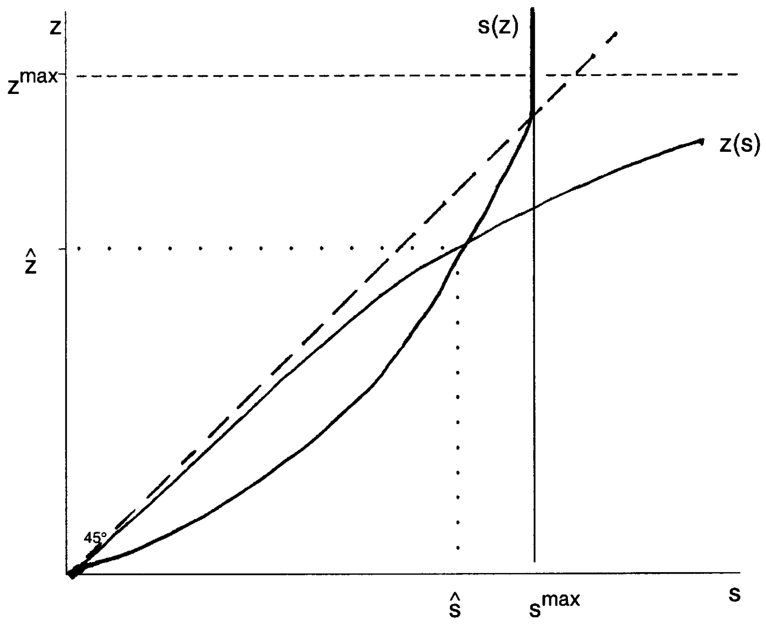

FIGURE 2. First-order-plus-symmetry condition.

Corollary 1. *If the SOC is fulfilled then necessarily* $\mathrm{d}^2C(\hat{z})/\mathrm{d}\hat{z}^2 > 0$.

Proof. This follows directly from claim 5 and the fact that $p > 0$, $f[\hat{z}+1] > 0$ and $\hat{z} > 0$. □

Thus, a necessary condition concerning the production technology is that there are decreasing returns to scale, at least locally. Note that whether the SOC will actually be fulfilled depends also upon the strategy $\hat{t}$ for which the FOC$^+$s are fulfilled. The economic meaning of the sign of the second derivative of the cost function is clear, but, *a priori*, it does not seem to make much sense to make further assumptions concerning the shape of the $C(z)$ function, *i.e.*, with respect to the third derivative. One can only observe that $f[\hat{z}+1]$ is bounded below 1, and that therefore the right-hand side of the equation of claim 5 approaches zero if $\hat{z}$ goes to infinity, implying that it might be more likely that the SOC is fulfilled when $\hat{z}$ is larger. Suppose that there were no trading uncertainty. That is, when firm i suppliedz units it would know that it would sell $\hat{z}$ units: $f[\hat{z}] = 1$. Then, $f[\hat{z}+1] = 0$, and the condition of claim 5 would be $\mathrm{d}^2C(\hat{z})/\mathrm{d}\hat{z}^2 > 0$, which is a rather familiar expression for models without trading uncertainty. Finally, one has to consider the payoff V_i to firm i. Clearly, if the expected profit when all firms choose strategy t is negative, firm i will prefer to stay inactive, and no strictly positive SNE will exist.

Proposition 2. *Necessary conditions for a SNE to exist are:* $n/N > k/p$, $n/m > -\ln(1 - \{\mathrm{d}C(0)/\mathrm{d}z\}/p)$, *and* $\mathrm{d}^2C(z)/\mathrm{d}z^2 > 0$ *for some* z.

Proof. See Claim 4 and Corollary 1. □

These conditions imply that n, p and $\mathrm{d}^2C(z)/\mathrm{d}z^2$ must be large enough, while N, m, k and $\mathrm{d}C(0)/\mathrm{d}z$ must be small enough. Sufficient conditions for existence of a SNE depend in a complicated way upon the parameter values. Following Judd (1995), this seems a typical case in which a numerical approach can help the theorist to discern and express patterns that an analytical approach would have difficulty with. Therefore, based on the analytical results presented above, we have done a numerical analysis in order to get more insight into the values of k, $C(.)$, p, $n(.)$, m and N for which the derived existence conditions for a SNE are satisfied. The details can be found in Vriend (1996). The findings are intuitively clear, and can be summarized as follows. A SNE will exist unless the costs of making a market and/or producing for the market are too high relative to the price p for each possible extent of the market at all levels of production. This may be due not only to the level of prices (p) and costs (k and $\mathrm{d}C/\mathrm{d}z$) as such, but also to too low a number of potential buyers (n) in the economy or to too high a number of competing firms (m).

Some characteristics of the SNE

In a SNE, the economy splits up into a number of possibly overlapping markets: each firm creates its own market of size s^*, and produces z^*. We will now analyze its characteristics. We first consider the effects of changes in the parameter values. Without giving all the straightforward analytical details, and observing that each firm's profitability depends upon the net gains per transaction and upon its trading opportunities, our findings can be summarized as follows. Higher net gains per transaction, *i.e.*, a higher price and/or lower production and signaling cost parameters, will, ceteris paribus, lead to a SNE with increased supply, market-making activity and expected profits. A lower number of competing firms or a higher number of potential customers will lead to higher expected profits, but the effect on the level of signaling and production activity is ambiguous. The point is that there are two opposing effects upon the $s(z)$ curve in Figure 2. On the one hand, if the probability of success of any given signal, $\Pr(s)$, increases, then the probability of success of an additional signal increases. However, on the other hand, the probability of success of all other signals also increases, implying that the firm may expect more visitors, and the probability that the firm would have at least one unit of the commodity left, $F[z-1]$, decreases. As a result, the probability that an additional visitor would be fruitful decreases[20]. Hence, in general, a decrease of trading uncertainty will lead to higher expected profits, but not necessarily to increased signaling and production activity. However, we at least know what happens when n/m approaches infinity. Each signal sent will lead to a consumer visiting its sender, and hence a situation of certainty is approached. Notice that this implies that constrained efficiency (given p) is reached. That is, interpreting signaling costs as part of the production costs, there is *'marginal cost pricing'* in the limit.

[20] In Vriend (1996) we present some numerical examples of SNE, illustrating these findings.

Claim 6. If $n/m \to \infty$ then the FOC$^+$s will be fulfilled for $\hat{t} \equiv (\hat{z} = \{z : \mathrm{d}C/\mathrm{d}z = p - k\}, \hat{s} = \hat{z})$.

Proof. See Vriend (1996). □

The next proposition concerns the size of the economy, asserting that the SNE and individual market outcomes are independent of the size of the economy as long as the proportions of types of agents, *i.e.*, firms and (interested) consumers, remain constant[21]. Hence, we could read the parameters of the model such that the number of agents N is countably infinite, while m and $n(.)$ are the fractions of firms and interested consumers in the population.

Proposition 3. *If the 'set of parameters'* $\{k, C(.), p, N, n(.), m\}$ *leads to a SNE* (z^*, s^*, V^*) *then the 'set of parameters'* $\{k, C(.), p, \alpha \cdot N, \alpha \cdot n(.), \alpha \cdot m\}$ *leads to exactly the same SNE* (z^*, s^*, V^*) *for any* $\alpha > 0$.

Proof. See Vriend (1996). □

In an analysis of market-making, it seems obvious to consider the issue of the extent of the market. Two phenomena that are considered to be related to each other in the literature, are *the division of labor* and *the extent of the market* (see *e.g.*, Smith, 1776). A measure of the division of labor could be the relative number of agents in the economy producing a given commodity. In our model, this would be the number of firms, m, for given N. A measure of the extent of the market could be the number of agents in the economy that is informed about the fact that the commodity is on the market. In our model, this would be measured by the aggregate number of signals sent, S. The proposition implies, for example, that it cannot be excluded that the aggregate market for a commodity shrinks when the number of firms specialized in producing the commodity increases.

Proposition 4. *The extent of the market (S) is a function of the division of labor (m), but the sign of* $\mathrm{d}S/\mathrm{d}m$ *is not determined a priori.*

Proof. See Vriend (1996), where we show that the number of signals sent by an individual firm, s, is a function of the parameter m, and that an increase in m may lead to a leftward or rightward shift of the $s(z)$ curve in Figure 2. This is caused by the two opposing effects mentioned above. On the one hand, the probability of success of an additional signal increases, but on the other hand, the probability of having at least one unit of the commodity left, $F[z\text{-}1]$, decreases. □

Now we consider the efficiency of the SNE. Clearly, the allocation mechanism as such is informationally inefficient. We focus upon efficiency given the trading and communication structure of the model. One of the attractive features of Walrasian models is that *'The Market'* is operated efficiently. A market is efficient if all mutually advantageous trades are carried out, which implies that one will not find rationed demanders and rationed suppliers at the same time (see *e.g.*,

[21] An additional condition is that the parameters remain such so as to allow for the Poisson approximations.

Benassy, 1982). In this sense the market outcome of a SNE is inefficient. In a SNE, the overall economy will not be orderly for each price, as there may be some buyers as well as some firms rationed at the same time[22,23]. This would seem to be a rather prominent characteristic of decentralized economies.

Proposition 5. *Prob* $[(x_i - z_i) \cdot (x_j - z_j) < 0] > 0$ *for each* $i \in B$, $j \in D$.

Proof. See Vriend (1996). Here we give an outline. Firm i's supply is $z_i = z^* > 0$, while the stochastic demand directed to it is represented by $f[q]$. Thus, the probability that firm i is rationed is equal to the probability that it will receive less than z^* buyers: Prob $[(x_i - z_i) < 0] = F[z^* - 1] > 0$ for each $i \in B$. Next, an interested consumer j has a unit demand and may visit only one firm. Buyers will be rationed when they do not receive any signal or when they visit a firm that has already sold out. In Vriend (1996) we show that both possibilities may occur with positive probabilities: Prob $[(x_j - z_j) > 0] > 0$ for each $j \in D$. Hence, given the SNE strategy profile $\mathbf{t}^*$, *ex ante* (*i.e.*, before the random distribution of signals by the firms, the random choices of firms by the buyers, and the random order of their visits have led to a specific realization), we have Prob $[(x_i - z_i) \cdot (x_j - z_j) < 0] > 0$ for each pair $i \in B$, $j \in D$. □

As rationing with respect to the consumers is all-or-nothing, from their point of view the probability to be rationed is a good measure of the performance of the economy. For firms, however, rationing is a quite *'natural'* affair. Therefore, we now consider another measure of efficiency concerning the firms. We compare the SNE with an equilibrium that would be in the joint interest of all firms. A Symmetric Cooperative Equilibrium (SCE) is a vector of strategies such that all firms choose the same strategy, and the sum of the payoffs of all firms is maximized. According to the following proposition, a SNE is not efficient from the firms' point of view in the sense that a better, *i.e.*, preferred by all firms, vector of strategies exists. However, each individual firm will have an incentive to deviate from the SCE strategy t^c. Notice that taking the number of signals s as fixed, increasing z_i decreases the expected revenue per unit for firm i, but has no strategic effect on the other sellers. Hence, the inefficient overproduction of goods in the SNE is in some sense driven by the overproduction of signals in the SNE, which follows from the fact that the signals are strategic substitutes. Moreover, as Corollary 2 asserts, consumers are worse off in a SCE.

Proposition 6. *The equilibrium strategy* t^* *of a SNE involves more communication and production, but lower expected profits, than the equilibrium strategy* t^c *of a SCE, i.e.,* $z^* > z^c$ *and* $s^* > s^c$*, while* $V^* < V^c$.

[22] Allowing for more visits per buyer would not change the picture. Each buyer dissatisfied in his first round might be more successful in his second or third round. As a result, given the level of signaling s, the probability of rationing will be lower for both firms and buyers.

[23] In this respect the model differs from some other models applying stochastic rationing (*e.g.*, Weinrich 1984), where it is assumed that markets are orderly, thereby implicitly assuming some kind of *'central lottery'*.

Proof. See Vriend (1996). Considering the appropriate FOCs, we show that the $z(s)$ curve in Figure 2 remains the same, since the production by the other firms, $\mathbf{z_i}$, does not enter firm i's payoff. But now, the revenue for firm i of sending one additional signal will be lower when all other firms also send one additional signal simultaneously. As a result, the new $s(z)$ curve will be at the left of the $s(z)$ curve in Figure 2, and the intersection of the $s(z)$ and $z(s)$ curves will occur at values z^c and s^c that are lower than z^* and s^*. That $V^* < V^c$ follows. □

Corollary 2. *Prob [cons. rationed| SNE] < Prob [cons. rationed| SCE].*

Proof. See Vriend (1996), where we show that this follows directly from the fact that the values for both z and s are lower in a SCE. □

4. Concluding remarks

The two essential features of a decentralized economy taken into account in our model are, first, that individual agents need some information about other agents in order to meet potential trading partners, which requires some communication or interaction between these agents, and second, that in general agents will face trading uncertainty. A number of problems concerning decentralized trade is related to these features, and may therefore be studied within this framework very well.

For example, one could analyze the existence of *central markets.* Let us suppose that firms may decide to *'cooperate'* physically in the market-making process. Instead of each firm selling its production in its own market, there may be one or more common, central markets or distribution points. Suppose that the technology of market-making is still the same (the central distributors sending signals giving the address of the distributor and the message that he sells the commodity), and that there are no additional costs of running a central market. When one assumes that each firm may sell in a common market in proportion to its contribution in the market-making costs, one can, for example, consider the non-cooperative solution concept of a Nash equilibrium. Each firm chooses a market to join and decides how much to contribute to the signaling activity, taking the choices of the other firms as given. This seems to capture the essential function of a central distributor.

The functioning of *middlemen* may also be analyzed within the framework of our model. Middlemen are not intrinsically interested in the commodity itself, *i.e.*, they belong to neither the firms nor the consumers. They buy from sellers, and sell to buyers. This description still allows for a number of middlemen functions (*e.g.*, reducing real transaction costs, reducing storing costs, forming a buffer between fluctuating demand and supply, speculation, etc.), but the distinguishing characteristic of middlemen is that they make a profit by taking into account the *'matching'* problem of the economy. Thus, they create markets by sending signals to establish contact with both firms and consumers.

The model would seem to be an interesting starting-point also for the study of *price-making* in a decentralized economy. The resulting market structure in a SNE is imperfectly competitive, although the commodity traded is homogeneous. Each firm signals to s agents, and in so doing it creates its own market. Thus, buyers might know that a firm finds as a maximum s alternative buyers in the market, and therefore buyers have some monopsony power. Each buyer on the other hand, can trade only with those firms from which he has received a signal. Thus, each firm may know that a buyer visiting him will know of only a limited number of alternative firms to visit, and therefore they will have some monopoly power. Finally, the model of decentralized trade proposed seems useful for a study of the phenomenon of *liquidity*: an asset being more liquid if it may be sold more cheaply and surely (see, Hahn, 1988), or problems like *effective demand failures*: some agents being unwilling to demand/supply more of one commodity because of uncertainty about their trading possibilities concerning another commodity (see Grandmont, 1988b).

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